Professional Exam Review
Fundamental Equations for Sound Bytes

QUANTITATIVE METHODS: SOUND BYTE 2

Effective Annual Rate:
EAR = $(1 + r)^m - 1.0$

Future Value:
FV = PV $(1 + r)^n$

Present Value:
$$PV = \frac{FV}{(1+r)^n} = FV/(1 + r)^n = FV\,(1 + r)^{-n}$$

Future Value of Ordinary Annuity:
$$PMT\left[\frac{(1+r)^N - 1}{r}\right] = PMT\,(FVIFA_{r,n})$$

Present Value of Ordinary Annuity:
$$PMT\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right] = PMT(PVIFA_{r,n})$$

Future Value of Annuity Due:
FV annuity due = $PMT(FVIFA_{r,n})(1+r)$

Present Value of an Annuity Due:
PV annuity due = $PMT(PVIFA_{r,n})(1+r)$

Present Value of Perpetuities:
PV = PMT/i

Holding Period Return:
HPR = $(P_1 – P_0 + D_1)/P_0$

Bank Discount Yield:
$$r_{BD} = \frac{D}{F} x \frac{360}{t}$$

Holding Period Yield:

$$\text{HPY} = \frac{P_1 - P_0 + D_1}{P_0}$$

Effective Annual Yield:

$\text{EAY} = (1 + \text{HPY})^{365/t} - 1$

Money Market Yield:

$R_{MM} = \text{HPY}\ (360/t)$

Arithmetic Mean:

$\mu = \Sigma X / N, \quad \overline{X} = \Sigma X / n$

Weighted Mean:

$\overline{X}_w = \Sigma w_i \times X_i$

Geometric Mean Return:

$R_G = [\, \Pi\, (1 + r_t)\,]^{1/T} - 1 = [\, \Pi\, R_t)\,]^{1/T} - 1$

Harmonic Mean:

$\overline{X}_H = 1 / [\, \{ \Sigma\ (1 / X\,)\, \} / n\,] = n / \Sigma\ (1/X$

Mean Absolute Deviation:

$$\text{MAD} = \frac{\sum \left| X - \overline{X} \right|}{n}$$

Variance for a Population:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

Standard Deviation for a Population:

$\sigma = \sqrt{\text{variance}} = (\text{variance})^{1/2}$

Variance for a Sample:

$$s^2 = \frac{\sum \left(X - \overline{X}\right)^2}{n - 1}$$

Standard Deviation for a Sample:

$s = \sqrt{\text{variance}} = (\text{variance})^{1/2}$

Sharpe Ratio:

$$\frac{r_p - r_f}{s_p}$$

Joint Probability:

$P(AB) = P(A \mid B) \times P(B)$

Total Probability Rule:

$P(A) = P(A \mid S)\, P(S) + P(A \mid S^C)\, P(S^C)$

Total Probability Rule for Expected Value:

$E(X) = E(X \mid S)\, P(S) + E(X \mid S^C)\, P(S^C)$

Expected Value:

$$\text{Expected value} = \sum_{i=1}^{n} p_i x_i$$

Covariance:

$Cov\,(R_i, R_j) = \Sigma\,[\,(R_i - E(R_i))\,(R_j - E(R_j))\,P(S)\,]$

Correlation:

$\rho(R_i, R_j) = Corr(R_i, R_j) = Cov(R_i, R_j)\,/\,\sigma(R_i)\,\sigma(R_j)$

Calculating Covariances from Correlations:

$Cov(R_i, R_j) = Corr(R_i, R_j)\,\sigma(R_i)\,\sigma(R_j)$

Joint Probability Function:

$Cov\,(R_i, R_j) = \Sigma\,\Sigma\;P(R_i, R_j)\,[R_i - E(R_i)]\;[(R_j - E(R_j)]$

Bayes' Formula:

$$P(A_1|B) = \frac{P(A_1) \times P(B|A_1)}{P(A_1) \times P(B|A_1) + P(A_2) \times P(B|A_2)}$$

Combination Formula Notation:

$${}_nC_r = \frac{n!}{(n-r)! * r!}$$

Permutation Formula Notation:

$${}_nP_r = \frac{n!}{(n-r)!}$$

ECONOMICS: SOUND BYTE 3

Price Elasticity of Demand:

$$\text{price elasticity of demand} = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in price}}$$

Cross Elasticity of Demand:

$$\text{cross elasticity of demand} = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in price of substitute or complement}}$$

Income Elasticity of Demand:

$$\text{income elasticity of demand} = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in income}}$$

Elasticity of Supply:

$$\text{elasticity of supply} = \frac{\%\ \text{change in quantity supplied}}{\%\ \text{change in price}}$$

Economic Profit:

= Total Revenues — (Opportunity costs)
= Total Revenues — Total Explicit Costs — Total Implicit Costs

CPI:

$$\frac{\text{CPI basket price at current prices}}{\text{CPI basket price at base period prices}} \times 100$$

Inflation Rate:

$(\text{CPI}_{\text{ending}} - \text{CPI}_{\text{beginning}}) / \text{CPI}_{\text{beginning}}$

Money Multiplier:

(1 + c) / (r + c)

M:

[(1 + c) / (r + c)] × B

Inflation:

[(Ending price level - Beginning price level)] / Beginning price level

FINANCIAL STATEMENT ANALYSIS: SOUND BYTE 4

Basic Earnings Per Share:

$$\text{Basic EPS} = \frac{\text{net income - preferred dividends}}{\text{weighted average number of shares outstanding}}$$

Diluted Earnings Per Share with Convertible Preferred Stock:

$$\text{Diluted EPS} = \frac{\text{net income}}{\left(\begin{array}{l}\text{weighted average number of shares outstanding} + \\ \text{new common shares that would have been issued at conversion}\end{array}\right)}$$

Diluted Earnings Per Share with convertible Debt:

$$\text{Diluted EPS} = \frac{\text{net income} + \text{after - tax interest on convertible debt - preferred dividends}}{\left(\begin{array}{l}\text{weighted average number of shares outstanding} + \\ \text{new common shares that would have been issued at conversion}\end{array}\right)}$$

Diluted Earnings Per Share with Stock Options:

$$\text{Diluted EPS} = \frac{\text{net income - preferred dividends}}{\left(\begin{array}{l}\text{weighted average number of shares outstanding} + \\ \text{new shares that would have been issued at option exercise -} \\ \text{shares the could have been purchased with cash proceeds from exercise}\end{array}\right)}$$

FCFF:
NI + NCC + interest (1 – Tax rate) – FCInv – WCInv

FCFF:
CFO + interest (1 – Tax rate) – FCInv

FCFE:
CFO – FCInv + Net borrowing – Net debt repayment

Formulas for Depreciation Expense (note *n* is the life of the asset):

- **Straight-line:**

$$\text{depreciation expense in Year i} = \frac{1}{n} \times (\text{original cost - salvage value})$$

- **Sum-of-years' digits:**

$$\text{depreciation expense in Year i} = \frac{(n - i + 1)}{SYD} \times (\text{original cost - salvage value})$$

where SYD = 1 + 2 + … + *n*

- **Double-declining balance:**

$$\text{depreciation expense in Year i} = \frac{2}{n} \times (\text{original cost - accumulated depreciation})$$

Average Age of a Firm's Assets:
Average age = Accumulated depreciation / Depreciation expense

Average Depreciable Life of a Firm's Assets:
Average depreciable life = Ending gross investment / Depreciation expense

CORPORATE FINANCE: SOUND BYTE 5

Net Present Value (NPV):

$$NPV = \sum_{t=1}^{n} CF_t / (1 + r)^t - \text{initial outlay}$$

Internal Rate of Return (IRR):

$$IRR = \sum_{t=0}^{n} CF_t / (1 + IRR)^t = 0, \text{ where t runs from 0 to n}$$

Average Accounting Rate of Return:
AAR = average net income / average book value

Profitability Index:
PI = 1 + [NPV / initial outlay]

Weighted Average Cost of Capital (WACC):
$WACC = w_d r_d (1 - t) + w_p r_p + w_e r_e$

Money Market Yield:
HPY (360/t) or $\frac{360 r_{BD}}{360 - t \times r_{BD}}$

Bond Equivalent Yield:
[(Face value – purchase price) / Purchase price] x [365 / # of days to maturity]

Cost of Equity: Use either the:

CAPM: $E(R_i) = R_F + ß_i * [E(R_M) - R_F]$ where:

$E(R_i)$ is the expected return on stock i;

R_F is the risk-free rate of interest;

$ß_i$ is the beta of stock i, a measure of its sensitivity to the market return; and

$E(R_M)$ is the expected return on the market.

Outside developed countries add a **country equity risk premium:**

$r_i = R_F + B_i*[E(R_M) - R_F + \text{Country Risk Premium}]$

or

Dividend discount model: $r_e = D_1 / P_0 + g$ where:

g is the growth rate of dividends, D_1 is next period's per share dividend, and P_0 is the current per-share price. If the company has to issue new stock to fund the project, make a flotation cost adjustment: $r_e = [D_1 / P_0*(1 - f)] + g$. Internal equity is cheaper than external equity because of flotation costs.

or

Bond yield plus: where a risk premium is added to the firm's bond yield.

Number of Days Receivables:
Accounts receivable / [Sales on credit / 365]

Number of Days Inventory:
Inventory / [Cost of goods sold / 365]

Number of Days Payable:
Accounts payable / [Purchases / 365]

Money Market Yield:
[(Face value – purchase price) / Purchase price] × [360 / # of days to maturity]

Bond Equivalent Yield:
[(Face value – purchase price) / Purchase price] × [365 / # of days to maturity]

Discount-Basis Yield:
[(Face value – Purchase price) / Face value] × [360 / # of days to maturity]

DuPont Analysis:

Return on Assets = $ROA = \frac{\text{Net Income}}{\text{Revenues}} \text{ x } \frac{\text{Revenues}}{\text{Average Total Assets}}$

= Net profit margin x Total asset turnover

$$ROE = \frac{\text{Net Income}}{\text{Revenues}} \text{ x } \frac{\text{Revenues}}{\text{Average Total Assets}} \text{ x } \frac{\text{Average Total Assets}}{\text{Average Shareholders' Equity}}$$

= Net profit margin x Total asset turnover x Leverage

PORTFOLIO MANAGEMENT: SOUND BYTE 6

Expected Return:

$$E(R_i) = \sum_{i=1}^{n} p_i \; E\left(R_i\right)$$

Expected Return on a Portfolio:

$$E(R_{port}) = \sum_{i=1}^{n} W_i \; E\left(R_i\right)$$

Variance:

$$\text{Variance} = \sigma^2 = \sum_{i=1}^{n} \left[R_i - E(R_{i)})\right]^2 p_i$$

Covariance:

$Cov_{i,j} = \Sigma \{[R_i - E(R_i)]\;[R_j - E(R_j)]\}$

Formula showing the relationship between the covariance of two securities and the correlation coefficient:

$$Corr_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j} \quad Cov_{ij} = Corr_{ij}\;(\sigma_i)(\sigma_j)$$

Standard Deviation:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov_{ij}}$$

$$i \neq j$$

CAPM:

$$E(R_i) = R_F R + (R_M - R_F R)\beta_i$$

where, β_i (**beta**) is defined as $Cov_{i,M} / \sigma_M^2$. *Beta* is a standardized measure of systematic risk.

ANALYSIS OF EQUITY INVESTMENTS: SOUND BYTE 7

Rate of Return:

(Ending Equity/Beginning Equity) – 1

Margin Call Price:

$P^* = \$D/[(1 - MMR\%^*) \times Q]$

Price-Weighted Index:

$$DJIA_t = \sum_{i=1}^{30} \frac{P_{it}}{D_{adj}}$$

Market-Value Weighted Index:

$$\text{MV Index}_t = \frac{\sum P_t x Q_t}{\sum P_{base} x Q_{base}} \text{ x Beginning MV Index Value}$$

The general form of the DDM is:

$$V = \frac{D_1}{(1+k_s)^1} + \frac{D_2}{(1+k_s)^2} + \cdots + \frac{D_n}{(1+k_s)^n}$$

Preferred Stock:

$$V = \frac{\text{Dividend}}{k_p}$$

Infinite Holding Period, Constant Growth Model or Gordon Growth Model:

$$P_0 = \frac{D_1}{(k_s - g)}$$

Earnings Multiplier:

$$P = \frac{D_1}{k_s - g} \quad \rightarrow \quad \frac{P}{E_1} = \frac{D_1 / E_1}{k_s - g}$$

ANALYSIS OF FIXED INCOME INVESTMENTS: SOUND BYTE 8

Absolute Yield Spread:

Yield on bond 1 - Yield on bond 2
Where bond 2 is considered to be the "benchmark" issue, and its yield is less than that on bond 1.

Relative Yield Spread:

(yield on bond 1 - yield on bond 2) / yield on bond 2

Yield Ratio:

Yield on bond 1 / Yield on bond 2

After-Tax Yield:

Bond's pre-tax yield × (1 - marginal tax rate)

Taxable-Equivalent Yield:

Bond's tax-exempt yield / (1 - marginal tax rate)

Price (Value) of a Coupon-Bearing Bond:

	Present val. of		Present value
=	the annuity of	+	of the bond's
	interest income		par value ($1,000)

NOTE: This valuation model can be used with both annual- and semiannual-pay bonds.

Price (Value) of a Zero-Coupon Bond:

Present value of the bond's par value

Current Yield:

Annual int. income / Mkt. price of the bond

Find a bond's YTM using the following bond valuation model:

Price (Value) of a Coupon-Bearing Bond:

Present value of		Present value
the annuity of	+	of the bond's
interest income		par value

- Given we know the current market price of the bond (the left-hand side of the equation), find the discount rate that will equate the present value of the bond's cash flow (the right-hand side of the equation), to its current market price.
- With modifications, same approach can be used to find YTC and YTP.

Annual C/F Yield:

$[(1+\text{monthly CFY})^6 - 1] \times 2$

Reinvestment Income:

Computed terminal value (see below) - all coupon payments - par value

Where: terminal value = the future value of the amount invested (i.e., the current market price of the bond), using the bond's YTM, or some other stipulated return, as the required rate of return.

BEY of an Annual-Pay Bond:

$2 \times [(1+\text{yld on annual-pay bond})^{1/2} - 1]$

Yield on an Annual-Pay Basis:

$[(1+\frac{\text{Yield on a bond-equivalent basis}}{2})^2 - 1]$

Pricing a Bond with Spot Rates:

Bond Price = $CF_1 / (1+Z_1) + CF_2 / (1+Z_2)^2 + \ldots + CF_n / (1+Z_n)^n$

Where: CF = the annual or semiannual coupon pmts over the life of the issue, CF_n = the final cash flow, made up of the last coupon payment and the principal payment, and Z = the annual or semiannual spot rates.

The General Equation for Finding a Forward Rate:

$l\ Ft = [(1 + Z_n)^n / (1+Z_m)^m] - 1$

Where: period **n** is one period longer than period **m**, and Z_n is the spot rate for the longer period, while Z_m is for the shortest period.

Effective Duration:

$\frac{\text{Price (i}\downarrow) - \text{Price (i}\uparrow)}{2 \times \text{Initial Price} \times \text{Chg. in Yld}}$

Where: Price (i ↓) = new price of the bond if rates fall by a given change in yield, price (i ↑) = new price of the bond if rates rise by a given change in yield, initial price = current price of the bond, and change in yield = the given change in yield used in the numerator to compute both of the new bond prices—entered as a decimal (e.g., 50 basis = 0.005).

Duration Effect:

% Change in Price = -1 × Duration × Change in Yield

Where: -1 is used to reflect the inverse relationship between bond prices and yields, duration is the measure as computed in above, and change in yield is a specific yield change (entered as a decimal) that's expected to occur in the immediate future.

Convexity Adjustment:

Convexity adjustment to the % change in price = Convexity × $(\text{Change in Yield})^2$

Where: Convexity = the bond's computed convexity measure (given), and Chg. in Yield is entered as a decimal.

DERIVATIVE INVESTMENTS AND ALTERNATIVE INVESTMENTS: SOUND BYTE 9

Payoff on a Forward Rate Agreement (FRA) contract:

$$\text{FRA Payoff} = \text{Notional Principal}\left[\frac{(\text{rate at expiration} - \text{FRA quoted rate})\left(\frac{\text{rate period}}{360}\right)}{1 + \text{rate at expiration}\left(\frac{\text{rate period}}{360}\right)}\right]$$

Today's Margin:

$\text{Today's margin} = \text{Yesterday's margin} \;+/-\; \text{Contract units}\,(\text{Today's price - Yesterday's price})$ where the +/- should be applied as a + for a long futures position and as a - for a short futures position.

The Payoff of a Call Option at Expiration:

$$C_T = \text{Max}\,(0, S_T - X)$$

The Payoff of a Put Option at Expiration:

$$P_T = \text{Max}\,(0, X - S_T)$$

The lower bound of a European Call:

$$C_E \geq \text{Max}\left(0, S_0 - X/(1+r)^T\right)$$

The lower bound of a European Put:

$$P_E \geq \text{Max}\left(0, X/(1+r)^T - S_0\right)$$

The lower bound of an American Call:

$$C_A \geq C_E \geq \text{Max}\left(0, S_0 - X/(1+r)^T\right)$$

The lower bound of an American Put:

$$P_A \geq \text{Max}\,(0, X - S_0)$$

European Put-Call Parity indicates that:

$$C_E + X/(1+r)^T = P_E + S_0$$

A Synthetic European Call can be constructed using the following portfolio:

$$C_E = P_E + S_0 - X/(1+r)^T$$

A Synthetic European Put can be constructed using the following portfolio:

$$P_E = C_E - S_0 + X/(1+r)^T$$

<u>The presence of cash flows being generated by the underlying asset revises the lower bounds on European call options to</u>:

$$C_E \geq \text{Max}\left\{0, \left[S_0 \text{ - PV}\left(\text{CF over option's life}\right)\right] \text{ - X}/\left(1 + r\right)^T\right\}$$

<u>The presence of cash flows being generated by the underlying asset revises the lower bounds on European put options to</u>:

$$P_E \geq \text{Max}\left\{0, \text{X}/\left(1 + r\right)^T \text{ - }\left[S_0 \text{ - PV}\left(\text{CF over option's life}\right)\right]\right\}$$

<u>The value of a property estimated using the income approach to real estate valuation is the present value of net operating income, which is projected as a perpetuity</u>:

$$\text{Appraisal Price} = \frac{\text{NOI}}{\text{Market Cap Rate}}$$, where

$$\text{Market Cap Rate} = \frac{\text{Benchmark NOI}}{\text{Benchmark Transaction Price}}$$